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Quantum decoherence from adiabatic entanglement with external one or a few degrees of freedom

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Abstract. Based on the Born-Oppenhemer approximation, the concept of adiabatic quantum entanglement is introduced to account for quantum decoherence of a quantum system due to its interaction with a large system of one or a few degrees of freedom. In the adiabatic limit, it is shown that the wave function of the total system formed by the quantum system plus the large system can be factorized as an entangled state with correlation between adiabatic quantum states and quasi-classical motion configurations of the large system. In association with a novel viewpoint about quantum measurement, which has been directly verified by most recent experiments [*e.g.*, S. Durr *et al.*, Nature **33**, 359 (1998)], it is shown that the adiabatic entanglement is indeed responsible for the quantum decoherence and thus can be regarded as a "clean" quantum measurement when the large system behaves as a classical object. By taking the large system respectively to be a macroscopically distinguishable spatial variable, a high spin system and a harmonic oscillator with a coherent initial state, three illustrations are presented with their explicit solutions in this paper.

PACS. 03.65.-w Quantum mechanics – 03.65.Bz Foundations, theory of measurement, miscellaneous theories (including Aharonov-Bohm effect, Bell inequalities, Berry's phase) – 03.65.Sq Semiclassical theories and applications

1 Introduction

In general, time-irreversible processes in quantum mechanics, such as the wave packet collapse of a measured system in quantum measurement and the quantum decoherence of a small system surrounded by an environment [1–3], suffer information loss due to the interaction of the considered system S with an external system E (the measuring instrument or the environment) *with many particles or many degrees of freedom*. In this paper we will show that, even if the external system has only one or a few degrees of freedom, it can still cause decoherence of the quantum system under the adiabatic condition. It turns out that the Born-Oppenhemer approximation leads to a partial factorization of the wave function of the total system formed by the quantum system plus the external large system and the motion configuration of the external system can then record the information of the quantum system effectively.

Physically, for Young's two-slit experiment the quantum decoherence is reflected by the disappearance of interference pattern in the presence of a "which-way" detector E. Associated with the wave-particle duality, before the measurement to observe which way the particle actually takes, the quantum particle seems to move from one point to another along several different ways simultaneously. This just reflects the wave feature of a quantum particle. The detection of "which-way" means a probe for the particle's feature, which leads to the disappearance of wave feature or quantum decoherence. The recent "which-way"experiments [4–7] show that Schrödinger's concept of entangled state, besides the unavoidable measurement distribution, is also crucial for the wave-particle duality. A quantum entangled state [8–10] such as

$$|\Psi\rangle = \sum_n C_n |S_n\rangle \otimes |D_n\rangle \quad (\neq |S\rangle \otimes |D\rangle \tag{1.1}$$

(for any $|S\rangle$ and $|D\rangle, C_n$ is a complex number) is a coherent superposition of states of the total system. It involves a correlation between the states $|S_n\rangle$ of the quantum system and the states $|D_n\rangle$ of E. Once the external system is found in a state $|D_n\rangle$, the total system must collapse into a certain component $|S_n\rangle \otimes |D_n\rangle$. Then one can infer the state $|S_n\rangle$ of the quantum system. The interference pattern can be described mathematically by using the

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reduced density matrix

$$\rho = \mathrm{Tr}_D(|\Psi\rangle\langle\Psi|) = \sum_n |C_n|^2 |S_n\rangle\langle S_n| + \sum_{m\neq n} C_m^* C_n |S_n\rangle\langle S_m| \langle D_m|D_n\rangle \quad (1.2)$$

which is obtained by tracing out the variables of E. The above-mentioned decoherence phenomenon can be equivalently expressed as a projection or reduction of the reduced density matrix from a pure state $\rho = \sum_{m,n} |S_n\rangle\langle S_m|$ to a mixed state $\hat{\rho} = \sum_n |S_n\rangle\langle S_n|$. The off-diagonal terms on the rhs of the above equation is responsible for the interference pattern. It is easy to see that the interference fringes completely vanish when the states of E are orthogonal to one another [10], *i.e.*, when $\langle D_m|D_n\rangle = \delta_{m,n}$. In this situation, an ideal quantum measurement results from the ideal entanglement with the correlated components $|D_n\rangle$ orthogonal to one another, in which one can distinguish the states of E very well.

It is noticed that, so long as the "which-way" information already stored in the detector *could be read out*, the interference pattern has been destroyed *without any data read out in practice* [4,5]. In this sense the environment surrounding the quantum system behaves as a detector to realize a "measurement-like" process. This is because the environment *never needs to read out* the data. Thus, the above argument is also applicable to the analysis of decoherence problem of an interfering quantum system coupling to the environment [8–10]. In this kind of problems, the environment is imagined as an objective detector detecting the states of the quantum system and thereby the detector states $|D_n\rangle$ are thought to be the macroscopic quantum states of the environment. Provide the environment couples with the quantum system and produce an ideal entanglement, the quantum system must lose its coherence. It is worthy to point out that this simple entanglement conserves the energy of the quantum system while destroying the quantum coherence. The loss of energy of the quantum system can be separately discussed in the quantum dissipation theory well developed in recent years [11–16].

In our previous works on quantum measurement theory [17–23], we investigate how an ideal entanglement appears in the macroscopic limit that the number N of particles making up the detectors approaches infinity. It was found that the *factorization structure*

$$F_{m,n} = \langle D_m|D_n\rangle \equiv \prod_{j=1}^{N} \langle D_m^{[j]}|D_n^{[j]}\rangle \quad (1.3)$$

concerning the overlapping of detector-states plays a crucial part in quantum decoherence. Here, $|D_n^{[j]}\rangle$ are the single states of those blocks constituting the detector, and $F_{m,n}$ is called decoherence factor. Since each factor $\langle D_m^{[j]}|D_n^{[j]}\rangle$ in $F_{m,n}$ has a norm less than unity, the product of infinite such factors may approach zero. This investigation was developed based on the Hepp-Coleman mode and its generalizations [24–27]. In 1998, this theory was applied to the analysis of the universality [28] of the influences environment [29] on quantum computing process [30,31]. Parallelly, the classical limit that certain quantum numbers (such as angular momentum) are huge is also investigated in our previous works.

However we have not got a totally-satisfactory answer to the question why the large system entangling with the small system behaves so classically in such limit situations. In fact, concerning the transition of the detector from quantum status to classical status, there were only some vague presentations [17,19,21] in the cases with large quantum number. In a general situation the classical feature of the large system can not simply be characterized by large quantum numbers, and thus what is responsible for the classical feature remains unclear yet. Besides, all of our previous discussions about quantum decoherence are based on interaction of particular forms, namely the non-demolition interaction [3]. In this paper, using Born-Oppenheimer (B-O) approximation [32], we universally consider the decoherence problem for a quantum system coupling to a large system with one or a few degrees of freedom through a most general interaction. This basic approach can be applied to analyzing influences exerted by environment and detector as well. Our discussion is also involved with a fundamental problem that the physicist can not avoid completely: how does the time reversal symmetry implied by the Schrödinger equation at the microscopic scale turn into the time reversal asymmetry manifested by quantum decoherence or quantum dissipation at the macroscopic scale?

This paper is organized as follows. We describe in Section 2 the adiabatic factorization of slow and fast dynamic variables in terms of the B-O approximation and show how the interaction of the large object of one or a few degrees of freedom with a quantum system causes a quantum entanglement dynamically. In Section 3, incorporating the semiclassical approach to the quasi-classical motion of slow variable in a smooth potential, we manifest that, driven by the adiabatically-effective Hamiltonian, the final states of the large object initially in an appropriate state are orthogonal to one another,and their entanglement with the quantum system leads to decoherence. In Sections 4, 5 and 6, the universal formal treatment in Sections 2 and 3 is illustrated by three explicit examples:

a. a particle with spin 1/2 moving slowly in an inhomogeneous magnetic field of varying direction (it is similar to the Stern-Gerlach experiment [33]);
b. a two level quantum system interacting with a very large spin (it is the generalization of the Cini model [34]);
c. a quantized cavity field coupled with a simple harmonic oscillator.

The third illustration has certain practical significance as it is relevant to the problem of detecting gravitational wave by intracavity dynamics [35,36].

2 Quantum entanglement via Born-Oppenheimer approach

In a very wide sense, any interaction between two quantum systems can cause an entanglement between them. In general, it then realizes a quantum measurement in a certain meaning. This is because one quantum system in different states can act on another with different effects correspondingly. However, this entanglement and its relevant quantum measurement is generally not very ideal because the usual interaction can not produce a one-one correspondence between the states of the two systems. Indeed, only a very particular interaction or its effective reduction can lead to an ideal entanglement and thereby an ideal quantum measurement. Nevertheless, fortunately, so long as one of the two systems can be separated *adiabatically* and behaves *classically*, as we will prove in the following, any interaction can result in an ideal entanglement in the evolution of the total system through its adiabatic reduction based on Born-Oppenheimer (B-O) approximation.

From the view point of B-O approximation, we consider a total quantum system ("molecular") with two sets of variables, a fast ("electric") one q and a slow (nuclear) one x. Resolving the dynamics of fast variables for a given motion of the slow subsystem, we obtain certain quantum states labeled by n for the fast part. To the first order approximation, the left effective Hamiltonian governing the slow variables involves an external scalar potential $V_n(x)$ and an magnetic-like vector potential $A_n(x)$ induced by the fast variables [37,38]. The latter is called the induced gauge potential or Berry's connection. If we assume the motions of the slow subsystem are "classical", we naturally observes that, due to the back-actions of the fast part, there are different induced forces

$$F_n = -\nabla_x V_n(x) + \frac{\mathrm{d}}{\mathrm{d}t} x(\nabla_x \times A_n) \tag{2.1}$$

exerting on the slow part. Their direct physical effects are that the information of the "fast" states labeled by n is recorded in the different motion configurations of the slow part. An entanglement just stems from this correlation between the quantum states of the fast subsystem and the classical motion configurations of the slow subsystem. In spirit of this physically-intuitive observation, we study the production of such quantum entanglement from the adiabatic separation of slow and fast variables based on the B-O approach.

Let us consider the interaction between a quantum system S with fast dynamic variable q and the large system E with slow variable x. The former with the Hamiltonian $H_\mathrm{s} = H_\mathrm{s}(q)$ can be regarded as a subsystem soaked in an environment or a measured system monitored by a detector, and the latter with the Hamiltonian $H_\mathrm{E} = H_\mathrm{E}(x)$ as the environment or the detector accordingly. In general the interaction Hamiltonian is written as $H_\mathrm{I} = H_\mathrm{I}(x, q)$. For a fixed value of slow variable x of E, the dynamics of the quantum system is determined by the eigen-equation

$$[H_\mathrm{s}(q) + H_\mathrm{I}(x, q)]|n[x]\rangle = V_n(x)|n[x]\rangle. \tag{2.2}$$

Both the eigen-values $V_n[x]$ and the eigen-state $|n[x]\rangle$ depend on the slow variable x as a given parameter.

Usually, the variation of the Hamiltonian $H_\mathrm{s}(q) + H_\mathrm{I}(x, q)$ with as a parameter x can cause transition from an energy level $V_n(x)$ of the quantum system to another level $V_m(x)$. But within the spatial domain R to which the slow variable x belongs, if the variable x changes so slowly that the adiabatic conditions [39–42]

$$\left|\frac{\langle n[x]|\partial_x|m[x]\rangle \mathrm{d}x/\mathrm{d}t}{V_m(x) - V_n(x)}\right| = \left|\frac{\langle n[x]|\{\partial_x H_I(x, q)\}|m[x]\rangle \mathrm{d}x/\mathrm{d}t}{\{V_m(x) - V_n(x)\}^2}\right| \ll 1 \tag{2.3}$$

hold for any two of the different energy levels $V_n(x)$, this transition can be physically neglected and then the B-O approximation works as an effective approach. Let $|\Phi_{n,\alpha}\rangle$ be the full eigen-function of the full Hamiltonian $H = H_\mathrm{E}(x) + H_\mathrm{s}(q) + H_\mathrm{I}(x, q)$ for the total system formed by the large system plus the quantum system. The B-O approximation treats it as a partially factorized function

$$\langle x|\Phi_{n,\alpha}\rangle = \phi_{n,\alpha}(x)|n[x]\rangle \tag{2.4}$$

of the slow and fast variables x and q. Here, the set of slow components $\{\phi_{n,\alpha}(x) = \langle x|\phi_{n,\alpha}\rangle\}$ and the corresponding eigen-values $\omega_{n,\alpha}$ are obtained by solving the effective eigen-equation

$$H_n(x)\phi_{n,\alpha}(x) = \omega_{n,\alpha}\phi_{n,\alpha}(x). \tag{2.5}$$

The effective Hamiltonian $H_n(x)$ is defined by

$$H_n(x) = H_{n\mathrm{E}}(x) + V_n(x) \tag{2.6}$$

where $H_{n\mathrm{E}}(x)$ is an gauge-covariant modification of $H_\mathrm{E}(x)$. It was obtained by replacing the momentum operator $p = -\mathrm{i}\hbar\nabla_x$ with its gauge-covariant form $p = -\mathrm{i}\hbar\nabla_x - A_n(x)$. Here, $A_n(x) = \mathrm{i}\langle n[x]|\nabla_x n[x]\rangle$ is a $U(1)$ gauge potential induced by the motion of the quantum system. In the classical limit that the slow part behaves classically, an effective dynamics of interaction between quantum and classical objects naturally results from the effective Hamiltonians or its relevant Lagrangian [43].

The completeness relations $\sum_{n,\alpha}|\Phi_{n,\alpha}\rangle\langle\Phi_{n,\alpha}| = 1$ for the full eigen-functions $|\Phi_{n,\alpha}\rangle$ can be expressed in *x-representation* as

$$\sum_{n,\alpha}\int \mathrm{d}x\mathrm{d}x'\phi*_{n,\alpha}(x')\phi_{n,\alpha}(x)|x\rangle\langle x'|\otimes|n[x]\rangle\langle n[x]| = 1 \tag{2.7}$$

which is equivalent to

$$\sum_n |x\rangle\langle x|\otimes|n[x]\rangle\langle n[x]| = |x\rangle\langle x|, \quad \sum_\alpha |\phi_{n,\alpha}\rangle\langle\phi_{n,\alpha}| = 1. \tag{2.8}$$

After obtaining the complete set $\{\phi_{n,\alpha}(x)|n[x]\rangle\}$ of eigenstates of the total system, we can now consider how the

entanglement appears in the adiabatic dynamic evolution. Let the total system be initially in the state $|\Psi(t=0)\rangle$:

$$\langle x|\Psi(t=0)\rangle = \sum_n c_n |n[x]\rangle \phi(x). \tag{2.9}$$

The first component of the initial state $|\Psi(t=0)\rangle$ is a superposition of the eigenstates of the quantum system while the second one a single pure state. Expanding $|\Psi(t=0)\rangle$ in terms of the complete set $\{\phi_{n,\alpha}(x)|n[x]\rangle\}$, we have the evolution wave function at time t

$$\langle x|\Psi(t)\rangle = \sum_{n,\alpha} c_n \langle \phi_{n,\alpha}|\phi\rangle \exp[-\mathrm{i}\omega_{n,\alpha}t]|n[x]\rangle \phi_{n,\alpha}(x) \tag{2.10}$$

where we have used the completeness relation equation (2.7). In terms of the effective Hamiltonian $H_n(x)$ related to each single adiabatic state $|n[x]\rangle$, the above wave function is rewritten in a concise form

$$\langle x|\Psi(t)\rangle = \sum_n c_n |n[x]\rangle \langle x|D_n(t)\rangle \tag{2.11}$$

with

$$|D_n(t)\rangle = \sum_\alpha \langle \phi_{n,\alpha}|\phi\rangle \mathrm{e}^{-\mathrm{i}\omega_{n,\alpha}t}|\phi_{n,\alpha}\rangle = \exp[-\mathrm{i}H_n t]|\phi(x)\rangle. \tag{2.12}$$

The full wave function$|\Psi(t)\rangle$ is obviously an entangled state. Starting from the same initial state $|\phi\rangle$ at $t=0$, the large system will be subject to different back-actions defined by (V_n, A_n) from the different adiabatic states $|n[x]\rangle$ of the quantum system. Then it evolves to a superposition of different final states $|D_n(t)\rangle$. This intuitive argument shows us that, there indeed exists an entanglement between two quantum systems with an quite general interaction, if one of them moves so slowly that their dynamic variables can be adiabatically factorized according to the B-O approximation. Roughly speaking, in the B-O approach, the slow subsystem is usually referred to as heavy particles (such as nucleons) while the fast one as light particles (such as the electrons). So it is reasonable to expect the slow subsystem to behave as a classical object.

3 Decoherence: Transition from quantum to classical

In this section we will discuss under what conditions the large system, the environment or the detector, can behave classically so that the quantum system entangled with it could completely lose its coherence and approach the classical limit.

Consider the reduced density matrix of the quantum system

$$\rho_s(t) = \mathrm{Tr}_D(|\Psi(t)\rangle\langle\Psi(t)|) = \sum_n |C_n|^2 |n[x]\rangle\langle n[x]| + \sum_{n\neq m} C_m C_n^* |m[x]\rangle\langle n[x]| \langle D_n(t)|D_m(t)\rangle \tag{3.1}$$

obtained by "summing over" the variables of the large system. The off-diagonal term responsible for interference is proportional to the overlapping $F_{n,m} = \langle D_n(t)|D_m(t)\rangle$ of the two large system states. Were there no large system interacting with it, the quantum system would be completely coherent for $\rho_\mathrm{s}(t) = |\varphi(t)\rangle\langle\varphi(t)|$ is a pure state. Here $|\varphi(t)\rangle = \exp(-\mathrm{i}H_\mathrm{s}t)|\varphi\rangle$ is a free evolution state of the large system. Mathematically, the effect of the adiabatic effective interaction is to multiply the off-diagonal term of the reduced density matrix by the decoherence factor $F_{n,m}$. A complete decoherence is defined by $F_{n,m}=0$ while a complete coherence by $F_{n,m}=1$ $(m\neq n)$.

Before considering how the decoherence factor $F_{n,m}$ becomes zero for the large system, we need to review some known arguments about the meaning of the classical limit of the motion of the large system. According to a widely accepted viewpoint [44], in the classical limit, the expectation value of an observable for certain particular states should recover its classical value forms. These particular states can give definite classical trajectories of particle in this limit. Usually we call them quasi-classical states. A coherent state or its squeezed version is a typical example of such states. According to Landau and Lifshitz [44], in general, a quasi-classical state is a particular superposition $\sum_n c_n\phi_n$ with the non-zero coefficients c_n only distributing around a large quantum number $\tilde{n}$. Then the correspondence principle requires that $\tilde{n}\to\infty$, $\hbar\to 0$ and the product $\tilde{n}\hbar$ approaches a finite classical action. In such a limit, the expectation of an observable will take the Fourier series of its corresponding classical quantity; or strictly speaking, it takes the Fejér's arithmetic mean of the partial sums of the Fourier series [45]. In this sense the mean-square deviation of the observable is zero; and accordingly the mean of the position operator defines a classical path. Physically, the zero mean-square deviation of the position operator implies the zero width of each wave packet $\langle x|D_m(t)\rangle$, and the overlapping $F_{n,m} = \langle D_n(t)|D_m(t)\rangle$ of zero width wave packets must vanish. From such a semi-classical picture, we will clearly see in the following how the decoherence factor $F_{n,m}$ approaches zero dynamically as the large system becomes classical.

In the semi-classical approach, for a heavy particle, the initial state $|\varphi\rangle$ can be regarded as a very narrow wave packet of width a. Since the heavy particle has a large mass M it hardly spreads in the evolution because without the environment induced quantum dissipation [15,16] the width of the wave packet at time t is

$$w(t) = a\sqrt{1+\frac{t^2}{4M^2a^4}}\,. \tag{3.2}$$

Then we describe the large system as an moving wave packet with the center along a classical path $x(t)$ on a manifold with local coordinates x. For a proper initial state $|\varphi\rangle$, we will see that the wave packet will split into several narrow peaks with the centers along different paths determined by different motion equations governed by the effective forces $F_n = -\nabla_x V_n(x) + (\mathrm{d}x/\mathrm{d}t)(\nabla_x \times A_n)$ with effective potentials $(V_n(x), A_n)$. Usually, the widths

of these peaks are almost of the same order as that of the original wave packet and each peak is correlated to an adiabatic quantum state $|n[x]\rangle$ for a large mass. Except for some moments at which the centers of two or more peaks coincide, these narrow peaks hardly overlaps with one another. In this sense, the large system starting from a narrow initial state can reach a superposition of those states orthogonal to one another. Thus we approximately have $F_{n,m} = 0$ in the classical limit for $m \neq n$.

With reference to the useful analysis in reference [10], we present an explicit but sketchy calculation to justify the above physically-intuitive observation about $F_{n,m} = 0$ in the classical limit. Assume the large system to be a heavy particle with very large mass M. In the duration τ of the adiabatic interaction with the quantum system, if the condition $v\tau \approx (\Delta p/M)\tau \ll \Delta x$ holds, the momentum p_0 of the free heavy particle can not be changed notably. Thus the contributions of the kinetic term and the induced gauge potential can be ignored in the wave function evolution of the free heavy particle under this condition. From this consideration we can approximately write down

$$|D_n(t)\rangle = \mathrm{e}^{-\mathrm{i}H_n t}|\phi(x)\rangle \propto \mathrm{e}^{-\mathrm{i}V_n(x)t}|\phi(x)\rangle. \tag{3.3}$$

The approximation requires that the effective potential $V_n(x)$ is satisfactorily smooth or the interaction $H_{\mathrm{I}}(x,q)$ is a smooth function of x. So we can use

$$V_n(x) \approx V_n(0) + F_n x;\ F_n \approx \nabla V_n(0) \tag{3.4}$$

to re-express the decoherence factor

$$F_{n,m} = \langle\varphi|\exp\Big(-\mathrm{i}t\delta F(m,n)x\Big)|\varphi\rangle. \tag{3.5}$$

Here, $\delta F(m,n) = F_m - F_n$ is the difference of two external forces exerted by two adiabatic potentials $V_m(x)$ and $V_n(x)$. Then the role of the back-action of the quantum system on the large system is summing up the momentum shift by a quantity $\delta F(m,n)t$ with respect to the initial state $|\varphi\rangle$. Obviously, when the width $\sigma = a^{-1}$ of the initial wave packet $\langle p|\varphi\rangle$ in the momentum space is much less than the momentum shift $\delta F(m,n)t$, the large system will adiabatically evolves into states orthogonal to one another. In fact, if the initial state is chosen to be a Gaussian wave packet $\langle x|\varphi\rangle = \sigma\exp[-\sigma^2x^2/2]/\sqrt{\pi}$ of width $\Delta x = 1/\sigma$, the decoherence factor is a Gaussian decaying function of time t

$$F_{n,m} = \exp\left(-\frac{\delta F(m,n)^2}{4\sigma^2}t^2\right). \tag{3.6}$$

As the evolution time t approaches infinity or if we have a very narrow width σ, $F_{n,m} \to 0$ and a quantum decoherence results from the dynamical evolution automatically.

Generally, we consider a system described by $H_n = p^2/2M + V_n(x)$ without the induced gauge field. Define $x_{\mathrm{c}} = \langle\varphi|x|\varphi\rangle$ and $p_{\mathrm{c}} = \langle\varphi|p|\varphi\rangle$ for an initial state $|\varphi\rangle$. In the classical regime one may expect that the variations $\xi = x - x_{\mathrm{c}}$ and $p_\xi = p - p_{\mathrm{c}}$ are small compared with x_{c} and p_{c}. Accordingly the potential can be expanded as

$$V_n(x) \simeq V_n(x_{\mathrm{c}}) + V_n'(x_{\mathrm{c}})\xi + \frac{1}{2}V_n''(x_{\mathrm{c}})\xi^2. \tag{3.7}$$

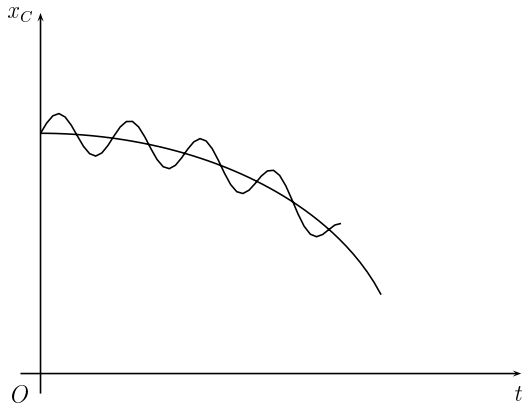

Fig. 1. Classical orbit with quantum fluctuation.

So, approximately the Heisenberg equations of motion become

$$\frac{\mathrm{d}}{\mathrm{d}t}x = \frac{p}{M}, \quad \frac{\mathrm{d}}{\mathrm{d}t}p = -V_n'(x_{\mathrm{c}}) - V_n''(x_{\mathrm{c}})\xi. \tag{3.8}$$

Sandwiched by the initial state $|\varphi\rangle$, the above equations turn into the classical equations of motion

$$\frac{\mathrm{d}}{\mathrm{d}t}x_{\mathrm{c}} = \frac{p_{\mathrm{c}}}{M}, \quad \frac{\mathrm{d}}{\mathrm{d}t}p_{\mathrm{c}} = -V'(x_{\mathrm{c}}). \tag{3.9}$$

Now we turn to Schrödinger's picture. The evolution of the initial state is governed by $\mathrm{i}\hbar\partial_t|\varphi(t)\rangle = H_n|\varphi(t)\rangle$. Introduce the following time-dependent translation

$$|\phi(t)\rangle = \exp\left\{\frac{\mathrm{i}}{\hbar}\Big(\theta(t) + x_{\mathrm{c}}p_\xi - p_{\mathrm{c}}\xi\Big)\right\}|\varphi(t)\rangle \equiv S(t)|\varphi(t)\rangle \tag{3.10}$$

where $\theta(t)$ is determined by $\dot\theta_t = p_{\mathrm{c}}^2/2M + V(x_c)$. Then straightforward calculation gives

$$\mathrm{i}\hbar\partial_t|\phi(t)\rangle = \left(\frac{p_\xi^2}{2M} + \frac{1}{2}M\omega_t^2\xi^2\right)|\phi(t)\rangle \tag{3.11}$$

where $M\omega_t^2 = V''(x_{\mathrm{c}})$. This exactly describes an oscillator with time-dependent frequency. The above direct derivation shows that in the non-inertial frame moving along the classical orbit, every quasi-classical system looks like a time-dependent oscillator whose frequency depends on the orbit. This fact is an established conclusion and illustrated in Figure 1. Actually, it is present in many textbooks about path integral. But our argument here is based on a clear physical picture and is applicable to the three dimensional case after a slight generalization.

Denote by $|0\rangle$ the vacuum state of the harmonic oscillator with frequency ω_0 which is equivalent to a Gaussian wave packet of width $\sigma_0^{-1} = \sqrt{2m\omega_0/\hbar}$. Suppose that initially the system is in the state $S^\dagger(0)|0\rangle$, a coherent state whose center lies at $(x_{\mathrm{c}}(0),\, p_{\mathrm{c}}(0))$. At time t, the center of the wave packet is obviously at $(x_{\mathrm{c}}(t),\, p_{\mathrm{c}}(t))$, and it is reasonable to expect that the width of the wave packet becomes $\sigma_t^{-1} = \sqrt{2m\omega_t/\hbar}$, since the frequency of the time-dependent oscillator changes very slowly. For two different

potentials $V_1(x)$ and $V_2(x)$ the macroscopic distinguishibility is ensured when the width sum of the two evolved packets is less than their orbital difference, that is, when

$$\sigma_{1t} + \sigma_{2t} \leq |x_{c1}(t) - x_{c2}(t)|.$$

One cannot expect that this condition can always be fulfilled for all time t. The orbital difference is determined by something like $|V_1' - V_2'|$ and the width is determined by the second derivative of the potential. But their relation is not very clear to us at present. What is clear is, to have $\langle D_1(t)|D_2(t)\rangle = 0$ one should require the points that fail the inequality form a zero measure set. On the other hand, the adiabatic approximation also imposes some restrictions on the potential. To clarify the situation further more sophisticated considerations are needed.

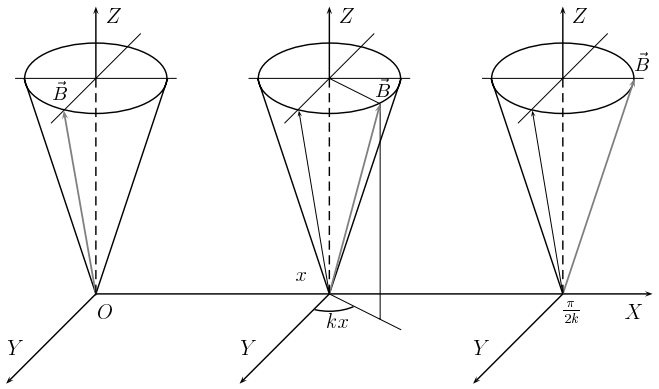

Fig. 2. The configuration of a rotating magnetic field for Stern-Gerlach experiment.

4 From macroscopic distinguishibility to decoherence

In the context of quantum measurement, a variant of the Stern-Gerlach (S-G) experiment provides an illustration of the above formalism. Quantum measurement is mutationally an observing process that "reads out" the system states from the "macroscopically distinguishable" states of the detector. As is shown in the above, if the large particle moves slowly enough, an adiabatic eigen-state of the quantum system will be correlated to one of the detector states in the B-O approximation. So the adiabatic correlation

$$|1[x]\rangle \to |D_1(t)\rangle,\ |2[x]\rangle \to |D_2(t)\rangle,\ ...\,|n[x]\rangle \to |D_n(t)\rangle \tag{4.1}$$

between the system states $|n[x]\rangle$ and the detector states $|D_n\rangle$ defines a quantum measurement. In the classical limit, this measurement is thought to be ideal for $|D_n\rangle$ $(n = 1, 2, ...)$ are orthogonal to one another, *i.e.*, $|D_n\rangle$ are shown to be "classically- or macroscopically distinguishable". Once the detector is found in the state $|D_n\rangle$, we can infer that the system is just in the state $|n\rangle$. In the following we will quantitatively analyze the dynamical realization of such an adiabatic measurement in a variant of the S-G experiment.

The original Stern-Gerlach (S-G) experiment can be considered as a quantum measurement process detecting the spin states of particles from their spatial distribution. The WPC or quantum decoherence can be described in an dynamical evolution governed by the interaction between the space- and spin-degrees of freedom. In its variant, a spin-1/2 particle initially in a certain superposition state enters an inhomogeneous magnetic field of amplitude $B(x)$ with *varying* direction $\mathbf{n}(x) = (\sin\theta\cos kx, \sin\theta\sin kx, \cos\theta)$ where θ is fixed. Its configuration is shown in Figure 2. A simple experiment though it is, it is among the candidates of experiments proposed to test the Berry's phase or its corresponding induced gauge field for a neutron in a static heliacal magnetic field [42,46]. In the usual S-G experiment, the direction of the magnetic field is along the fixed x-axis, but in our present model the *polarization direction varies* as the position x changes.

The spatial variable is considered to be the slow system while the spin-variable to be fast as a quantum system. Corresponding to the eigenvalues $V_\pm(x) = \pm\mu B(x)$, the adiabatic eigenstates of the spin-Hamiltonian $H_{\rm spin} = \mu B(x)\,\mathbf{n}(x)\cdot\boldsymbol{\sigma}$ are

$$|\chi_+[x]\rangle = \begin{bmatrix} \cos\theta/2\ \mathrm{e}^{-\mathrm{i}kx} \\ \sin\theta/2 \end{bmatrix}, \quad |\chi_-[x]\rangle = \begin{bmatrix} \sin\theta/2\ \mathrm{e}^{-\mathrm{i}kx} \\ -\cos\theta/2 \end{bmatrix}.$$

Here $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ is the Pauli spin operator and μ the gyromagnetic ratio. Let the incoming beam be initially in a superposition of the adiabatic eigen-states $|\psi\rangle = c_+|\chi_+[x]\rangle + c_-|\chi_-[x]\rangle$ along a certain polarization direction depending on x. When the particle moves so slowly that the adiabatic condition

$$\left|\frac{\mathrm{d}}{\mathrm{d}t} xk\sin\theta/\mu B(x)\right| \ll 1 \tag{4.2}$$

holds, to the lowest order of the B-O approximation, the total initial state $|\Psi(0)\rangle = \{c_+|\chi_+[x]\rangle + c_-|\chi_-[x]\rangle\}\otimes|\phi(x)\rangle$ will evolve into an entangled state

$$|\Psi(t)\rangle = c_+|\chi_+[x]\rangle \otimes |D_+(t)\rangle + c_-|\chi_-[x]\rangle \otimes |D_-(t)\rangle. \tag{4.3}$$

Here, $|D_\pm(t)\rangle = \exp[-\mathrm{i}H_\pm t]|\phi(x)\rangle$ are the spatial states governed by the effective Hamiltonians

$$H\pm = \frac{1}{2M}(-\mathrm{i}\partial_x - A_\pm)^2 + V_\pm(x). \tag{4.4}$$

The effective scalar potentials $V_\pm(x)$ and the induced vector potentials $A_\pm = k(1 \pm \cos\theta)/2$ are determined from the adiabatic spin eigenstates $|\chi_+[x]\rangle$ and $|\chi_-[x]\rangle$. In the semi-classical picture, because the particles in the adiabatic spin states $|\chi_+[x]\rangle$ and $|\chi_-[x]\rangle$ separately suffer two forces $F_\pm = -\partial V_\pm(x)/\partial x$ of opposite directions along $\mathbf{x}$, they will finally form two macroscopically-distinguishable

spots on the detecting screen, each of which is correlated to one of the spin states. This spin-space correlation process enables people to pick out different spin states according to the spatial distribution.

To analyze this measurement process in details we assume the spatial part $\phi(x)$ in the initial state is a Gaussian wave packet

$$|\phi(x)\rangle = \left(\frac{1}{2\pi a^2}\right)^{\frac{1}{4}} \int \mathrm{d}x\, \mathrm{e}^{-\frac{x^2}{4a^2}}\, |x\rangle \tag{4.5}$$

distributing along direction x with the center at the original point. Here a is the initial width of the atom beam. Adopting the semi-classical method, we have the linear approximation $B(x) \simeq [\partial_x B\,(x=0)]x$ and $f = \mu\partial_x B\,(x=0)$. Factorizing the evolution operator $U_\pm(t) = \exp[-\mathrm{i}H_\pm t]$ by Wei-Norman method [47,48], we exactly obtains, in position representation, the following effective wave functions $|D_\pm(t)\rangle$at time t

$$\langle x|D_\pm(t)\rangle = \left(\frac{a^2}{2\pi^3}\right)^{\frac{1}{4}} \left(\frac{\pi}{a^2+\frac{\mathrm{i}t}{2M}}\right)^{\frac{1}{2}} \mathrm{e}^{-\mathrm{i}\Omega_\pm(t)\mp\mathrm{i}ftx} \times \exp\left[-\frac{(x-x_{\pm\mathrm{c}}(t))^2}{4(a^2+\frac{\mathrm{i}t}{2M})}\right] \tag{4.6}$$

where

$$\Omega_\pm(t) = \frac{f^2t^3}{6M} + \frac{1}{2}ft^2A_\pm.$$

It is seen from equation (4.6) that the Gaussian wave packets $\langle x|D_\pm(t)\rangle$ center on the classical trajectories

$$x_{\pm\mathrm{c}}(t) = \mp\frac{1}{2}\frac{f}{M}t^2 - \frac{A_\pm}{M}t. \tag{4.7}$$

They have the different group speeds $v_\pm = \mp ft/M - A_\pm/M$ along the opposite directions, but have the same width $a(t) = a\sqrt{1+t^2/(4M^2a^2)}$ spreading with time. It is obvious that the motions of the wave packet centers obey the classical motion law that a particle of mass M forced by $\mp f$ will move with the acceleration $\mp f/M$. The quantum character of this motion is mainly reflected in the spreading of the wave-packets. The induced gauge fields $A_\pm$ are constant, but they change the initial value of $\mathrm{d}x/\mathrm{d}t$ according to the corresponding classical Hamilton equation

$$M\frac{\mathrm{d}^2}{\mathrm{d}t^2}x = \mp f; \quad \frac{\mathrm{d}}{\mathrm{d}t}x = \frac{p}{M} - \frac{A_\pm}{M}\,. \tag{4.8}$$

This means that the zero initial value of the canonical momentum $p = M\mathrm{d}x/\mathrm{d}t + A_\pm$ determines the initial velocity $\mathrm{d}x(0)/\mathrm{d}t = -A_\pm/M$. The quantum effects of $A_\pm$ are to contribute the additional phases $-ft^2A_\pm/2$ in the wave functions.

The macroscopic distinguishibility of wave-packets in quantum measurement requires that the distance between the two wave-packets should be larger than the width of each wave packet, *i.e.*

$$ft^2 - k\cos\theta t \gg a\sqrt{M^2+\frac{t^2}{4a^2}}\,. \tag{4.9}$$

This condition is easily satisfied for a long time evolution.

To analyze the decoherence quantitatively, we compute the norm of the decoherence factor $F(t) = |\langle D_+(t)|D_-(t)\rangle|$. The extent of quantum coherence depends totally on this overlapping integral. We can explicitly integrate it

$$F(t) = \exp\left[-a^2f^2t^2 - \frac{1}{8a^2}\left(\frac{f}{M}t^2 - \frac{k\cos\theta}{M}t\right)^2\right]. \tag{4.10}$$

It is obvious that the decoherence process indeed happens as $t\to\infty$, but it does not obey the simple exponential law $\mathrm{e}^{-\gamma t}$. In a long time scale, the temporal behavior of decoherence is described by $F(t) \approx \exp\left[-f^2t^4/8a^2M^2\right]$ and the characteristic time of the decoherence process can be defined by $F(\tau_\mathrm{d}) = \mathrm{e}^{-1}$, that is

$$\tau_\mathrm{d} = \sqrt{\frac{2\sqrt{2}Ma}{f}}\,. \tag{4.11}$$

This shows that the long time behavior of decoherence is independent of the spatial details of interaction, which is caused by the configuration of the external field.

5 Decoherence resulting from large spin

There is a second illustration to show the happening of decoherence owning to the adiabatic separation of two systems. Based on our previous investigation about quantum decoherence in the classical limit [17,21,22], we assign an arbitrary spin j to interact with a two-level system (such as a spin-1/2 system) through a coupling of particular form. Let $\mathbf{J} = (\hat{J}_x, \hat{J}_y, \hat{J}_z)$ be the angular momentum operator of the large system and $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ be the Pauli matrix describing the quasi-spin of the two-level quantum system with energy-level difference ω_s. The full Hamiltonian of this model is

$$H_\mathrm{I} = \omega_\mathrm{s}\sigma_z + \omega J_z + f(\mathbf{J})\sigma_x. \tag{5.1}$$

The general interaction $f(\mathbf{J})\sigma_x$ is linear with respect to the variable of the quantum system while it depends on the variable $\mathbf{J}$ through a function $f(\mathbf{J})$. Two free Hamiltonians $\omega_\mathrm{s}\sigma_z$ and ωJ_z were introduced to consider the energy-exchange between the quantum system and the large system.

The interaction $f(\mathbf{J})\sigma_x$ can not well distinguish the states $|\pm 1/2\rangle$ of the quantum system for $|\pm 1/2\rangle$ are not the eigen-states of the interaction Hamiltonian. So, in general, this model can not well describe a quantum measurement process and thus can not give a good description of quantum decoherence. However, if we think $\mathbf{J}$

as the slowly-changing variable relative to the fast one σ, determined by the B-O approximation under the adiabatic condition, the effective potential $V_{\pm} = \pm\sqrt{\omega_{\rm s}^2 + f(\mathbf{J})^2}$ of the large system will clearly distinguish the adiabatic eigen-states $|u_+[\mathbf{J}]\rangle = (\cos\vartheta/2, \sin\vartheta/2)^T$ and $|u_-[\mathbf{J}]\rangle = (\sin\vartheta/2, -\cos\vartheta/2)^T$. Here, the angle parameter $\vartheta = \arctan(-f(\mathbf{J})/\omega_{\rm s})$ depends on the slow variable $\mathbf{J}$. Then, the adiabatic separation of the spin-1/2 and the large spin system will result in a quantum decoherence.

In fact, because of the introduction of the arbitrary spin j, which labels the $(2j+1)$-dimensional irreducible representation of the rotation group SO(3), we are able to consider the behaviors of the quantum dynamics governed by this model Hamiltonian in the classical limit with infinite spin j. The reason why the limit with infinite j is called classical is that the mean square deviations of the components $\hat{J}_x$, and $\hat{J}_y$ enjoy the following limit feature $\Delta\hat{J}_x/j = \Delta\hat{J}_y/j = 1/\sqrt{2j} \to 0$ as $j \to \infty$ [17,34,49].

To solve the dynamical evolution of the total system explicitly, we choose a particular form of interaction: $f(\mathbf{J}) = \sqrt{g^2 J_x^2 - \omega_{\rm s}^2}$. Taking this particular form is equivalent to making a linear approximation for the effective potential $V_{\pm}[\mathbf{J}]$. With this particular form the effective Hamiltonians $H_{\pm} = \omega J_z + V_{\pm}[\mathbf{J}]$ can be expressed as an rotation of the simple spin-Hamiltonian $H_0 = \sqrt{g^2+\omega^2}J_z$, *i.e.*,

$$H_{\pm} = \exp[\mathrm{i}\hat{J}_y\phi_{\pm}]H_0\exp[-\mathrm{i}\hat{J}_y\phi_{\pm}] \tag{5.2}$$

where the polar angle $\phi_{\pm}$ is defined by $\tan\phi_{\pm} = \pm g/\omega$.

According to the quantum angular momentum theory, the eigen-states of $H_{\pm}$ can be constructed as

$$|j, m(\phi_{\pm})\rangle = \exp[\mathrm{i}\hat{J}_y\phi_{\pm}]|j,m\rangle = \sum_{m=-j}^{j} d^j_{m',m,}(-\phi_{\pm})|j,m'\rangle \tag{5.3}$$

where $|j,m\rangle$ is a standard angular momentum state and $d^j_{m',m,}(\phi) = \langle j,m'|\exp[\mathrm{i}\hat{J}_y\phi]|j,m\rangle$ is the corresponding d-function; the corresponding eigen-values are $E_m = m\sqrt{g^2+\omega^2}$.

Here, we should remark that the exact solvability of the above model largely depends on the particular form of the function $f(\mathbf{J})$. If this is not the case, the above method can not work well and then certain semi-classical approximation methods should be used to deal with the effective Hamiltonian in its classical limit with very large j. If the coupling function $f(\mathbf{J})$ depends on $\mathbf{J}$ quite slightly, we can generally linearize the above effective potential $V_{\pm}(\mathbf{J})$ to realize the particular form.

We are concerned with classical characters of the large-spin system. Let us suppose it is initially assigned the adiabatic ground state $|j, m = -j(\phi)\rangle$ with the lowest magnetic quantum number $m = -j$. In quantum measurement theory, the choice of ground state is required by a stable measurement. Starting with its initial state

$$|\psi(0)\rangle = (C_+|u_+[\mathbf{J}]\rangle + C_-|u_-[\mathbf{J}]\rangle) \otimes |j, -j(\phi)\rangle \tag{5.4}$$

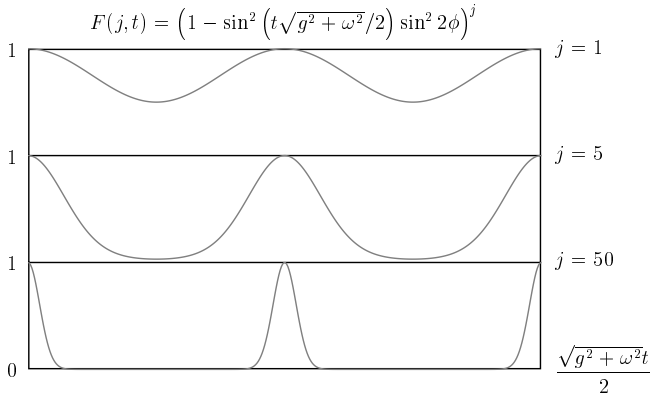

Fig. 3. Disappear of non-diagonal elements of density matrix. Here $\sin^2\phi = 1/2$.

the effective Hamiltonians (5.2) evolves the large spin system into an entanglement state

$$|\psi(t)\rangle = C_+|u_+[\mathbf{J}]\rangle \otimes |D_+(t)\rangle + C_-|u_-[\mathbf{J}]\rangle \otimes |D_-(t)\rangle, \tag{5.5}$$

with

$$|D_{\pm}(t)\rangle = \exp[\pm\mathrm{i}\hat{J}_y\phi]\exp[-\mathrm{i}t\overset{\wedge}{\mathbf{J}}_z\sqrt{g^2+\omega^2}] \times \exp[\mp\mathrm{i}\hat{J}_y\phi]|j, -j(\phi)\rangle. \tag{5.6}$$

Using the explicit expressions of the d-function $d^j_{m',m}(\phi_{\pm})$, we can calculate the overlapping $\langle D_-(t)|D_+(t)\rangle$, obtaining

$$F(j;t) = |\langle D_-(t)|D_+(t)\rangle| = \left|1 - \sin^2 2\phi \sin^2\frac{\sqrt{g^2+\omega^2}}{2}t\right|^j. \tag{5.7}$$

The above formula directly manifests the happening of quantum decoherence in the classical limit $j \to \infty$. In fact, in a nontrivial case with $\phi \neq 0$,

$$\left|1 - \sin^2\frac{t}{2}\sqrt{g^2+\omega^2}\sin^2 2\phi\right|$$

is usually a positive number less than 1. In the classical limit with $j \to \infty$, its jth power $|\langle D_-(t)|D_+(t)\rangle|$ must approach for $t \neq t_n \equiv 2nt/\sqrt{g^2+\omega^2}, n = 0, 1, 2...$ At those instances t_n, quantum coherence revivals as so-called quantum jumps (see Fig. 3). Then, as far as the present model is concerned, we reach the conclusion that, if the large spin system behaves classically, the decoherence can be dynamically realized for the entangled quantum system. In traditional quantum measurement, the detector was pre-required as a purely classical object to reduce the coherent superposition instantaneously. But now it is proved that the WPC occurs as the *quantum* detector moves slowly to approach the classical limit. This means in our treatment the detector is essentially still a quantum object. Thus it has the advantage of dealing with the problem of quantum measurement consistently within the framework of quantum theory.

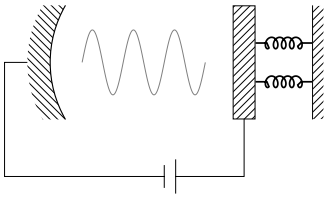

Fig. 4. Cavity with a oscillating mirror.

6 Intracavity dynamics with classical source

Our third example about decoherence in quantum adiabatic process is the intracavity dynamics with a classical source, which is associated with the interferometric detection of the gravitational wave by a squeezed light [35,36].

We consider a cavity with two end mirrors (as in Fig. 4), one of which is fixed while the other is treated as a simple harmonic oscillator of frequency Ω and mass M with the position and momentum x and p. The radiation pressure force of the cavity field on the moving mirrors is proportional to the intracavity photon density. Let $a^\dagger$ and a be the creation and annihilation operators of the cavity with a single mode of frequency ω. The cavity-mirror coupling is described by an interaction Hamiltonian $H_{\rm I} = gxa^\dagger a$ where g is the coupling constant depending on the electric dipole. In the radio frequency range the cavity field can be prescribed as a macroscopic current. From this consideration we describe the cavity field dynamics with the Hamiltonian $H_{\rm c} = \omega a^\dagger a + f(a^\dagger + a)$. This cavity field-mirror coupling system can also be used to detect the photon number in the cavity by the motion of mirror. Obviously, the motion of the mirror is slow with respect to the oscillation of the cavity field. Thus we can use the B-O approximation to approach the quantum decoherence problem in the measurement of the cavity field. Most recently, the special case of this model without classical source has been used as a scheme probing the decoherence of a macroscopic object [51].

Coupled with the mirror and the classical source, the adiabatic eigen-states

$$|n[x]\rangle = \frac{1}{\sqrt{n!}}[a^\dagger + \lambda(x)]^n|0\rangle \tag{6.1}$$

of the cavity field for displacement $\lambda(x) = f/(\omega + gx)$ are determined by

$$\{[\omega + gx]a^\dagger a + f(a^\dagger + a)\}|n[x]\rangle = v_n(x)|n[x]\rangle \tag{6.2}$$

with the corresponding eigen-values $v_n(x) = n(\omega + gx), n = 0, 1, 2, ...$ Under the B-O approximation, the effective Hamiltonians are also referred to as the forced harmonic oscillators in the same renormalization external potential (RNEP) $V_{\rm rne} = f^2/(\omega + gx)$ [11,12]. Under the adiabatic condition

$$\left|\frac{\langle(n-1)[x]\partial_x|n[x]\rangle\frac{\rm d}{{\rm d}t}x}{\omega + gx}\right| \sim \frac{|ngf\frac{\rm d}{{\rm d}t}x|}{\omega^3} \ll 1 \tag{6.3}$$

μ, the RNEP $V_{\rm rne}$ can be linearized as

$$\frac{f^2}{\omega}\left[1 - \frac{gx}{\omega}\right].$$

Then the effective Hamiltonians can be rewritten as $H_n = \Omega b^\dagger b + g_n(b^\dagger + b)$ in terms of

$$b = \frac{M\Omega\, x + ip}{\sqrt{2M\Omega}}, \quad g_n = \frac{g(n - f^2/\omega^2)}{\sqrt{2M\Omega}} = \mu\left(n - \frac{f^2}{\omega^2}\right). \tag{6.4}$$

For each effective Hamiltonian H_n, the corresponding evolution is a displacement operator

$$D[\alpha_n(t)] = \exp\left(\alpha_n(t)b^\dagger - \alpha_n(t)^* b\right) \tag{6.5}$$

with $\alpha_n(t) = -g_n(\exp[{\rm i}\Omega t] - 1)/\Omega$.

Let the initial state of the mirror be a well-defined quasi-classical state, a coherent state $|\alpha\rangle$ and the initial state of the cavity be a superposition $|c(0)\rangle = \sum_n c_n|n[x]\rangle$ of the adiabatic states. The evolution governed by the effective Hamiltonian H_n leads to an entangled state

$$\begin{aligned}|\psi_{\rm I}(t)\rangle &= \sum_n c_n|n[x]\rangle \otimes D[\alpha_n(t)]|\alpha\rangle \\ &\equiv \sum_n c_n|n[x]\rangle \otimes |D_n(t)\rangle\end{aligned} \tag{6.6}$$

for the total system. The overlapping of the mirror states in this entanglement can be computed and its norm is

$$|\langle D_m(t)|D_n(t)\rangle| = \exp\left(-(n-m)^2\frac{2\mu^2}{\Omega^2}\sin^2\frac{\Omega t}{2}\right). \tag{6.7}$$

The changing rate ${\rm d}x/{\rm d}t$ (the velocity) of the slow variable x is proportional to Ω. In the adiabatic limit, Ω is very small. So we can rationally consider the limit $\Omega \to 0$ for a fixed μ. Then an ideal entanglement appears in this limit case for the overlapping becomes an non-linear exponential decaying factor

$$|\langle D_m(t)|D_n(t)\rangle| = \exp\left(-\frac{1}{2}(n-m)^2\mu^2t^2\right). \tag{6.8}$$

This result is quite similar to that of the Cini model in van Hove limit [34]. This decay phenomenon was first illustrated in reference [21,23]. Mathematically, it results from the fact that in the strong coupling limit, the period of the oscillation is very large in comparison with the small frequency Ω.

Another interesting situation arise when the mirror is initially prepared in a Fock state $|n\rangle = \left(a^\dagger\right)^n|0\rangle/\sqrt{n}!$. To show a macroscopic, but non-classical dynamic behavior, the Fock state should possess a very large occupation number n. The overlapping for the initial Fock state can be expressed as

$$\begin{aligned}F(t,n) &= \langle n|D[-\alpha_k(t)]D[\alpha_l(t)]|n\rangle \\ &= \exp\left[-\frac{1}{2}(l-k)^2\frac{\mu^2}{\Omega^2}\sin^2\frac{\Omega t}{2}\right] \\ &\quad \times L_n\left((l-k)^2\frac{\mu^2}{\Omega^2}\sin^2\frac{\Omega t}{2}\right)\end{aligned} \tag{6.9}$$

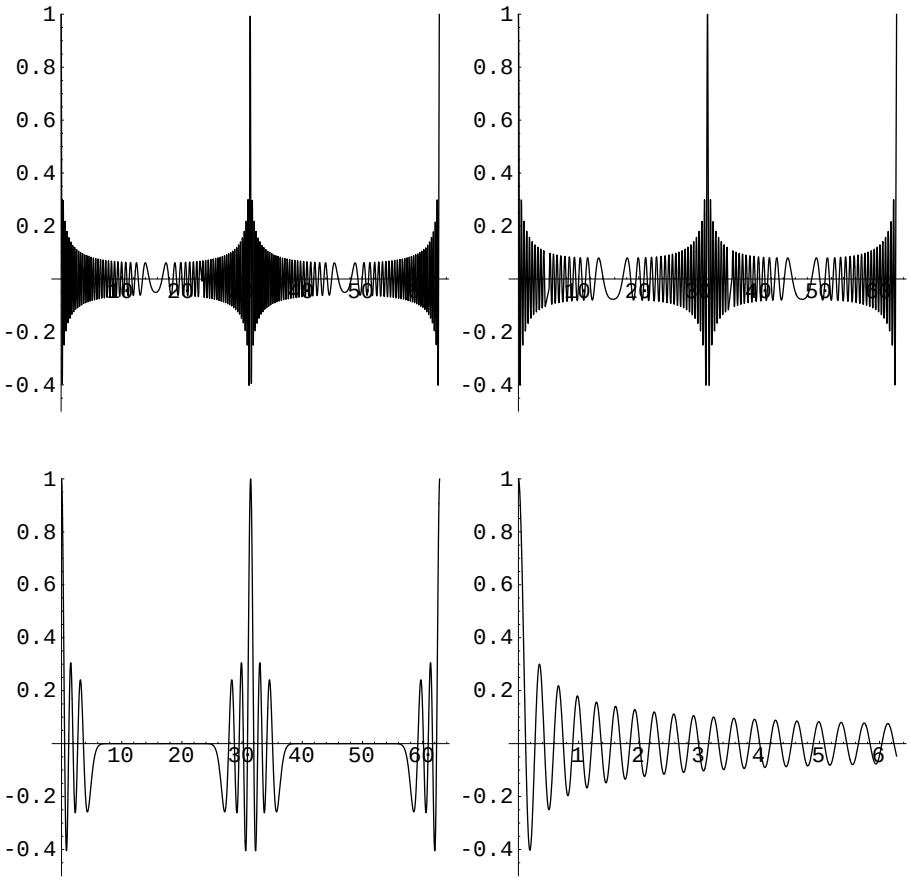

Fig. 5. Time-dependency of the nondiagonal elements.

in terms of the Laguerre polynomial $L_n(z)$. Figure 5 shows $F(t,n)$ as a function of time t for different j. In fact, according to the theory of special function, $L_n(z)$ approaches the zero-order Bessel function $J_0(\sqrt{n}z)$ when $n \to \infty$, hence [52],

$$F(t,n) \to \mathrm{e}^{-\frac{1}{2}(l-k)^2\mu^2t^2}\ L_n((l-k)^2\mu^2t^2/4) \qquad (6.10)$$

$$\to \mathrm{e}^{-\frac{1}{2}(l-k)^2\mu^2t^2}\ J_0(\sqrt{n}(l-k)^2\mu^2t^2). \qquad (6.11)$$

The zero-order Bessel function of real variable $\zeta\sqrt{n}$ is a decaying-oscillating function and approaches zero as n tends to infinity. Therefore, when the cavity is occupied by a large number of photons, the macroscopic feature of the detector (the end mirror) dynamically decoheres the initial pure state of the cavity.

7 Concluding remarks

In a wide sense, the adiabatic entanglement can be well understood in the picture of coupled channels [50], which is an extensive generalization of B-O approximation. Consider a total system whose wave function depends on two sets of variables, q and x. Let Q be an operator only acting on the function of q and has a complete set of eigen-vectors $\{|n\rangle\}$ with the corresponding eigenvalues v_n. Since $\{|n\rangle\}$ forms a complete basis of the Hilbert space of all functions of q, the total eigenfunction $\Psi_{\mathrm{E}}(x,q)$ of the full Hamiltonian $H = H_{\mathrm{E}}(x) + H_{\mathrm{s}}(q) + H_{\mathrm{I}}(x,q)$ with eigen-value E can be regarded as a function of q for a given x and then can be expressed as $\Psi_{\mathrm{E}}(x,q) = \sum \phi_n(x)|n\rangle$. The *channel wave function* $\phi_n(x)$ is defined by the *coupled channel equations*

$$H_{nn}(x)\phi_n(x) + \sum_{m\neq n} H_{nm}(x)\phi_m(x) = E\phi_n(x). \qquad (7.1)$$

The matrix elements $H_{mn}(x) = \langle m|H|n\rangle_q$ are defined in terms of the q-function space "integral". Under a certain condition, if the off-diagonal elements *can be neglected physically,* an effective non-demolition Hamiltonian $H_{\mathrm{eff}} = H_{\mathrm{E-eff}}(x) + H_{\mathrm{s-eff}} + H_{\mathrm{in}}(x)$:

$$\begin{aligned} H_{\mathrm{E-eff}} &= \mathrm{diag}[H^{\mathrm{E}}_{11}(x), H^{\mathrm{E}}_{22}(x), ..., H^{\mathrm{E}}_{dd}(x)], \\ H_{\mathrm{s-eff}} &= \mathrm{diag}[\lambda_1, \lambda_2, ..., \lambda_d], \\ H_{\mathrm{in}}(x) &= \mathrm{diag}[H^{\mathrm{s}}_{11}(x), H^{\mathrm{s}}_{22}(x), ..., H^{\mathrm{s}}_{d}(x)], \end{aligned} \qquad (7.2)$$

can be partially diagonalized in the "channel space". Here $H^A_{mm}(x) = \langle m|H_A|m\rangle_q$ for $A = E, S$ and $\lambda_m = \langle m|H_{\mathrm{s}}(q)|m\rangle_q$ *are constants.* Obviously, the non-demolition condition $[H_{\mathrm{s-eff}}, H_{\mathrm{in}}(x)] = 0$ holds as $H_{\mathrm{s-eff}}$ is a constant matrix. In the B-O approximation the channel operator Q is taken to be $Q[x] = H_{\mathrm{s}}(q) + H_{\mathrm{I}}(x,q)$,which is parameterized by x. The adiabatic condition maintains that, only the diagonal elements play a dominant role and the off-diagonal elements *can be neglected for very small channel-channel coupling* [42]. Therefore, it can be concluded that there may exist a more general mechanism beyond B-O approximation to

realize the quantum decoherence dynamically originating from the basic interaction, which is related to the theory of coupled channels.

Finally we point out that the presence of non-demolition interaction [3] is only a necessary condition for quantum decoherence to appear. Sufficient conditions should include the requirement that the large system be classical so that its final states could be orthogonal to one another. In this paper, we have regarded the spin-system with a very large spin and the harmonic oscillator initially in a coherent state as classical objects. Then within the semi-classical framework, even in the case of a general potential motion, we are able to relate the macroscopic distinguishibility of the quantum states of the large system to its classical limit behaviors. However, there are still vague points in the definition of the quantum-classical division for the large system. This problem is deeply rooted in the following more fundamental and more challenging issue: why or in what sense does a general large system behave classically. If we imagine that, besides the considered quantum system, there is another system coupling with the large system to decohere it, then the present problem will be trapped into an evil logic chain. One notices the difficulty here is very similar to that faced by von Neumann and Wigner about sixty years ago [1,2]. Though new experiments have been revitalizing the study of decoherence problem and progress is being made, it seems that there is still a long way to go to finally understand quantum irreversible process completely. To reach this goal; one should first find a satisfactory definition for the so called quantum-classical boundary. At present it is very unclear to us how to do this without recourse to particular physical systems.

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